ISDN2400 Physical Prototyping

Mechanisms

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Today's lecture

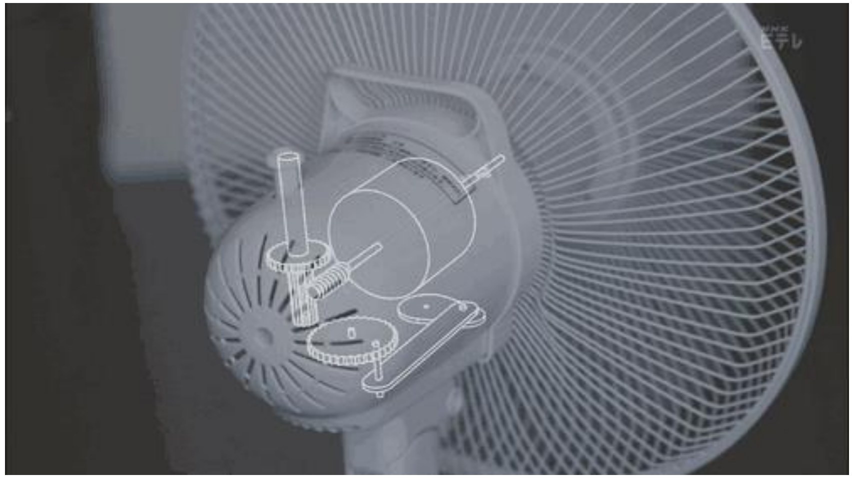
- Introduction to mechanisms
  - Gruebler's equation
  - Four-bar mechanisms
  - Mechanisms for converting rotary motion to reciprocating motion
- Time for teamwork

Last lecture & Lab

- Running a DC motor
- Motors are rotary: how to create the oscillatory motion of the fish tail?
  - Mechanisms

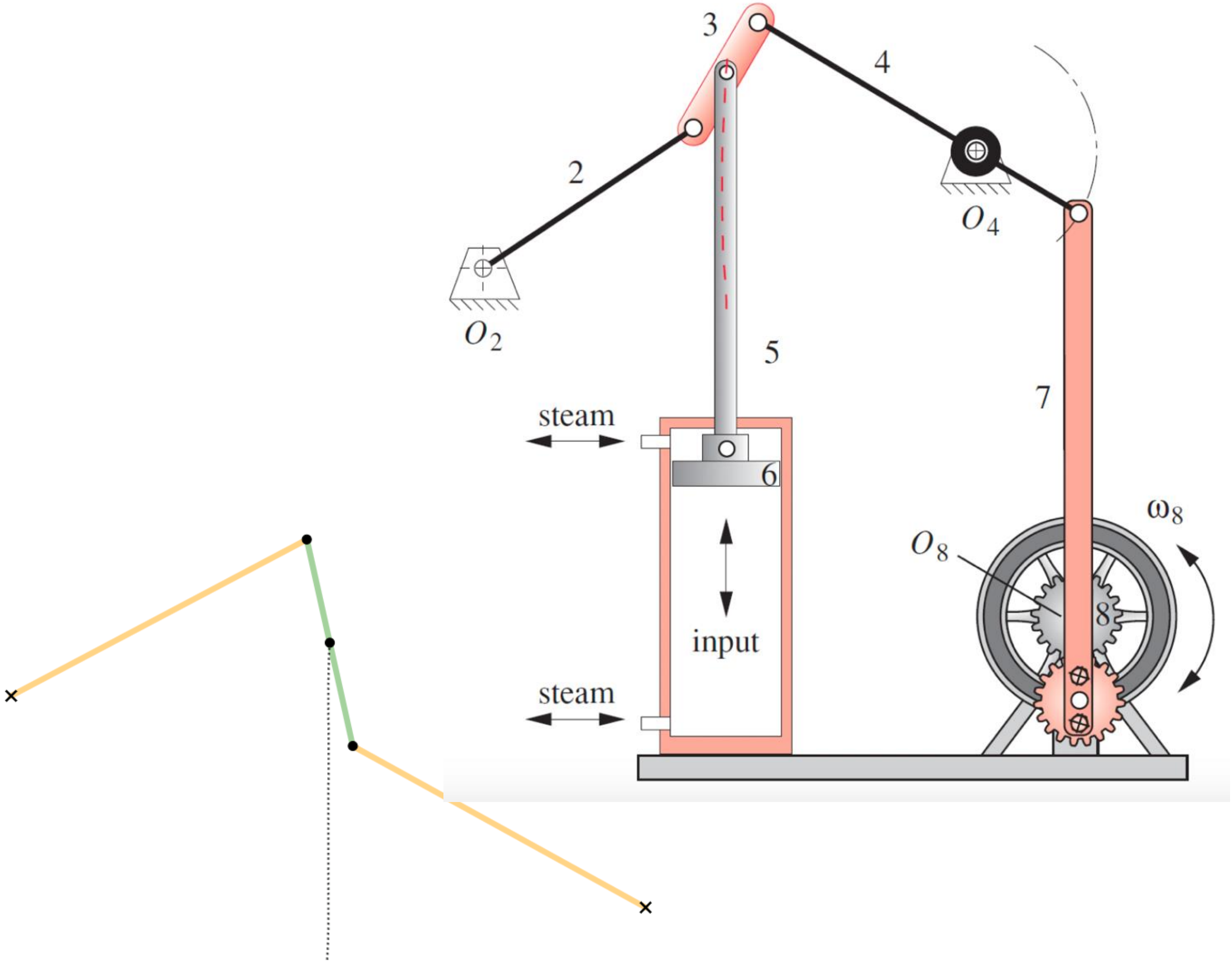
Mechanisms

- Everyday life examples?
- Mechanisms are everywhere
  - Not just robots, cars, and industrial machinery!


History of mechanisms

- Straight line motion was difficult to achieve
- First (approximate) straight line linkage invented by James Watt for guiding the piston of early steam engines
  - Patented in 1784

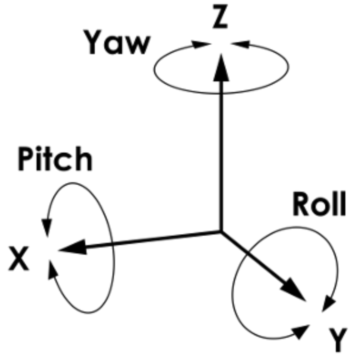


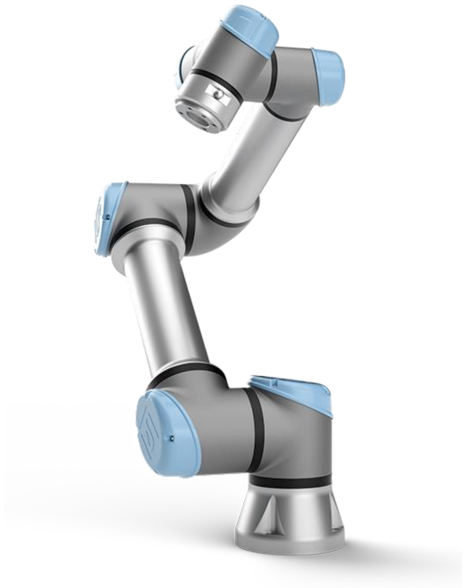
Definitions

- **Mechanism:** Any collection of parts that transforms input forces and movement into a desired set of output forces and movement
- **Linkage:** a group of bodies (or links) connected by joints
- **Kinematic chain:** subset of linkages with ideal joints (e.g. pure rotation or sliding) and rigid links, such that you can use geometry to relate the positions and velocities (kinematics) of each link to every other link in the chain

Degrees of Freedom

- **Degrees of Freedom (DOFs):** The number of independent variables that define the configuration or state of a mechanical system in space



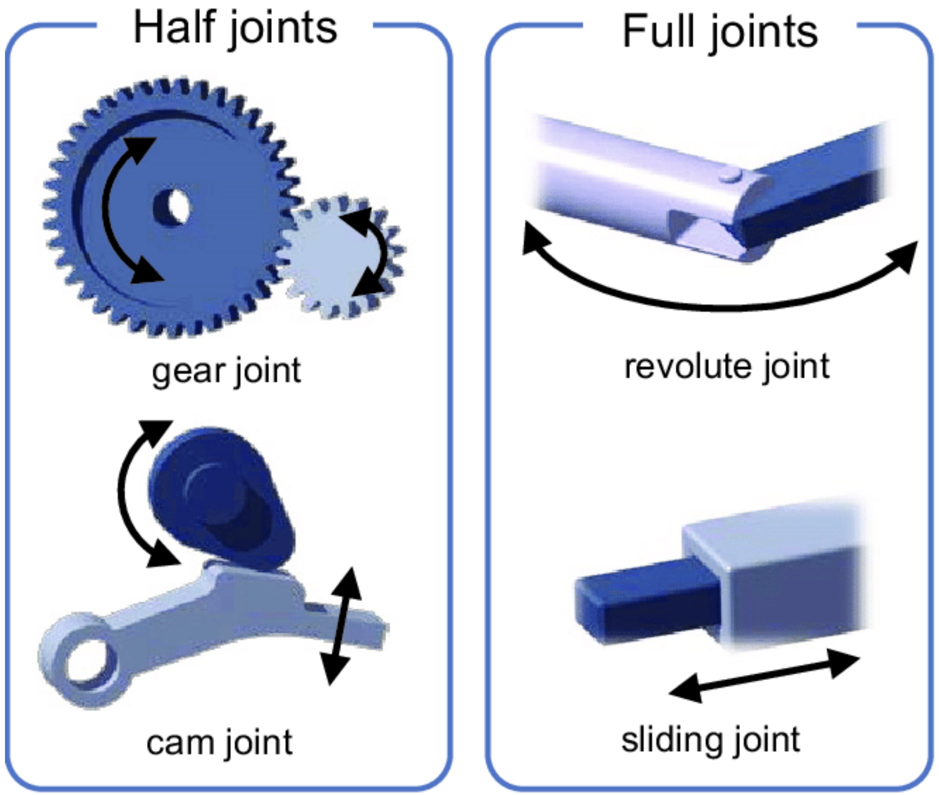


Kinematic chains

- How to determine the number of DOFs of a kinematic chain?
  - Gruebler's equation
- $F = 3(n - 1) - 2L - H$
  - F = number of degrees of freedom
  - n = total number of links in the mechanism
  - L = total number of lower pairs
  - H = total number of higher pairs

Joint types

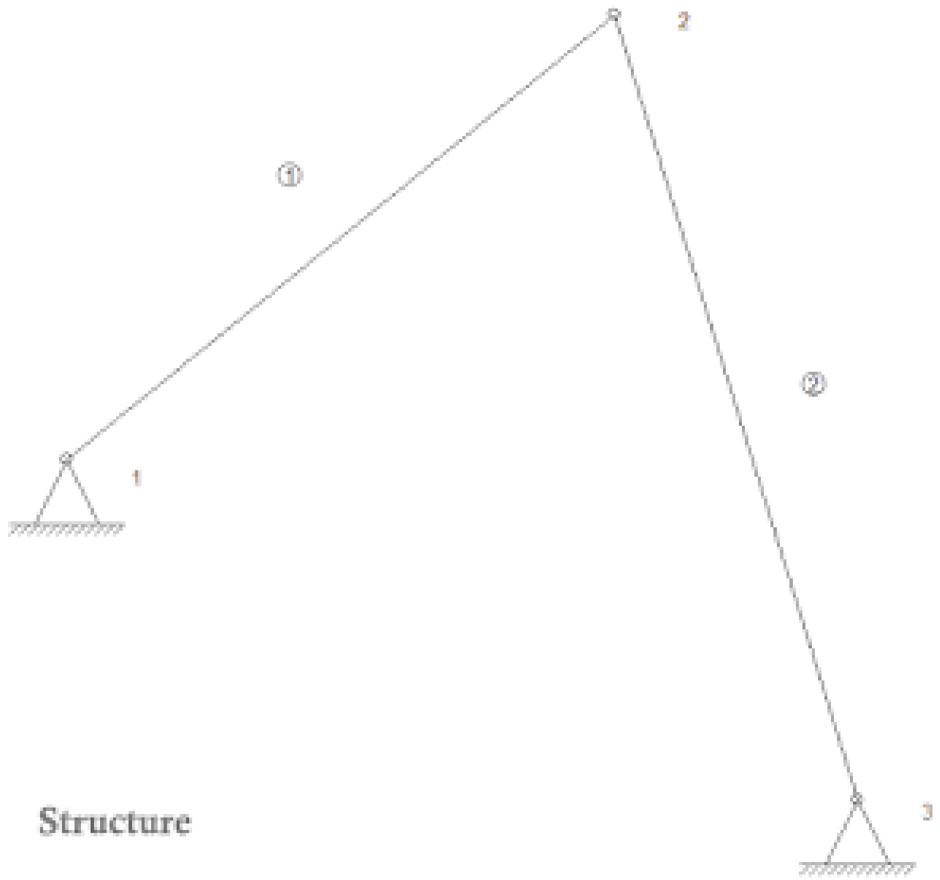
- Lower pairs
  - Surface contact between members
  - Also referred to as full joints
  - Revolute joints (pins) and prismatic joints (sliding joints)
- Higher pairs
  - Line or point contact between members
  - Also referred to as half joints
  - cam and gear joints



Heling et al., “Hybrid Tolerance Representation of Systems in Motion”

Gears

$$F = 3(n-1) - 2L - H$$

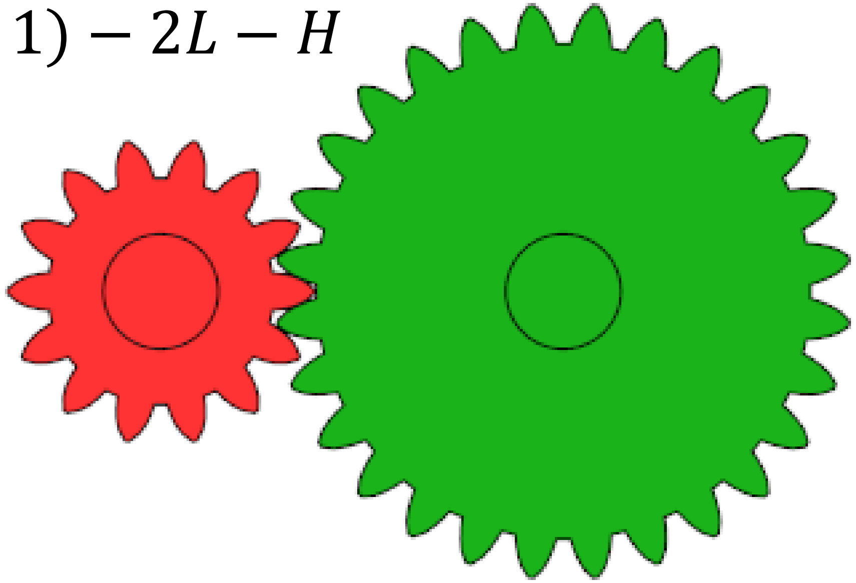


http://ezymechanic.blogspot.com/2015/07/gruebler-equation-for-calculating.html

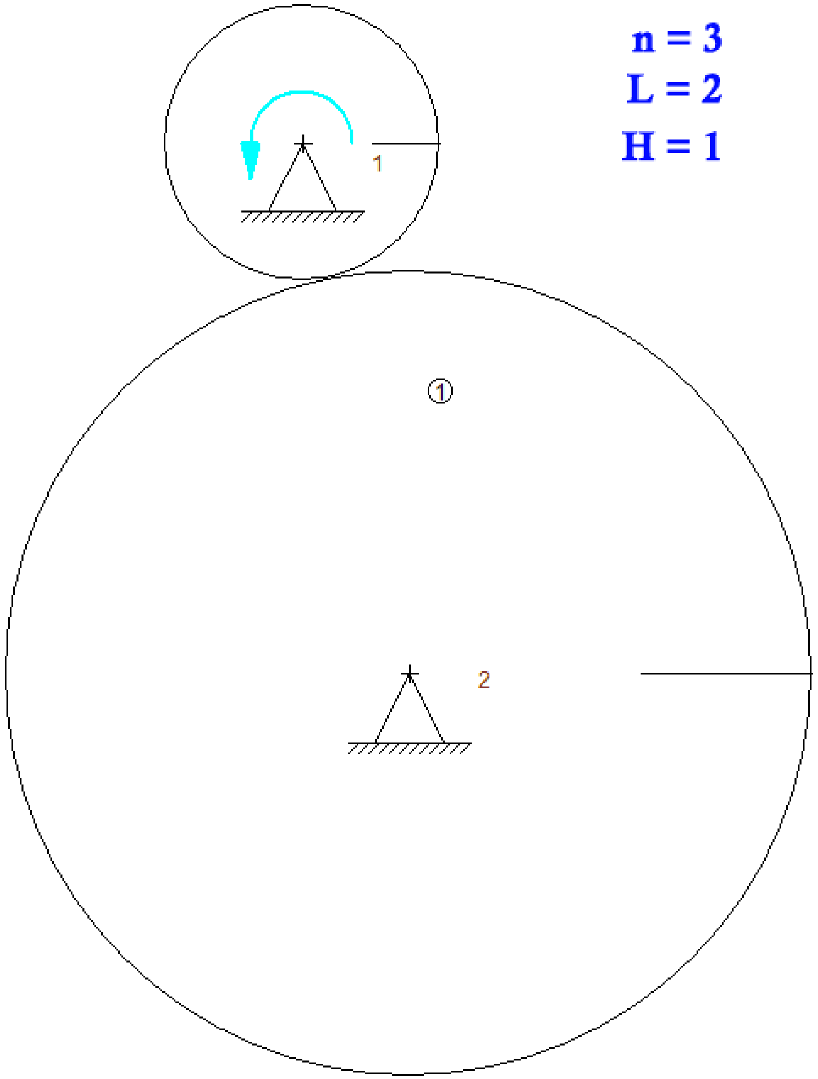
- $n = 3$
  - *2 links + 1 ground link*
- $L = 3$
  - *3 revolute joints*
- $H = 0$
- $F = 3(3-1) - 2(3) - 0 = 0$

Gears

$$F = 3(n-1) - 2L - H$$



- $n = 3$
  - *2 gears + 1 ground link*
- $L = 2$
  - *2 revolute joints*
- $H = 1$
  - *1 gear joint*
- $F = 3(3-1) - 2(2) - 1 = 1$

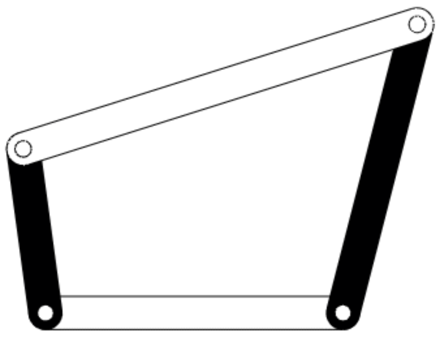


Gears

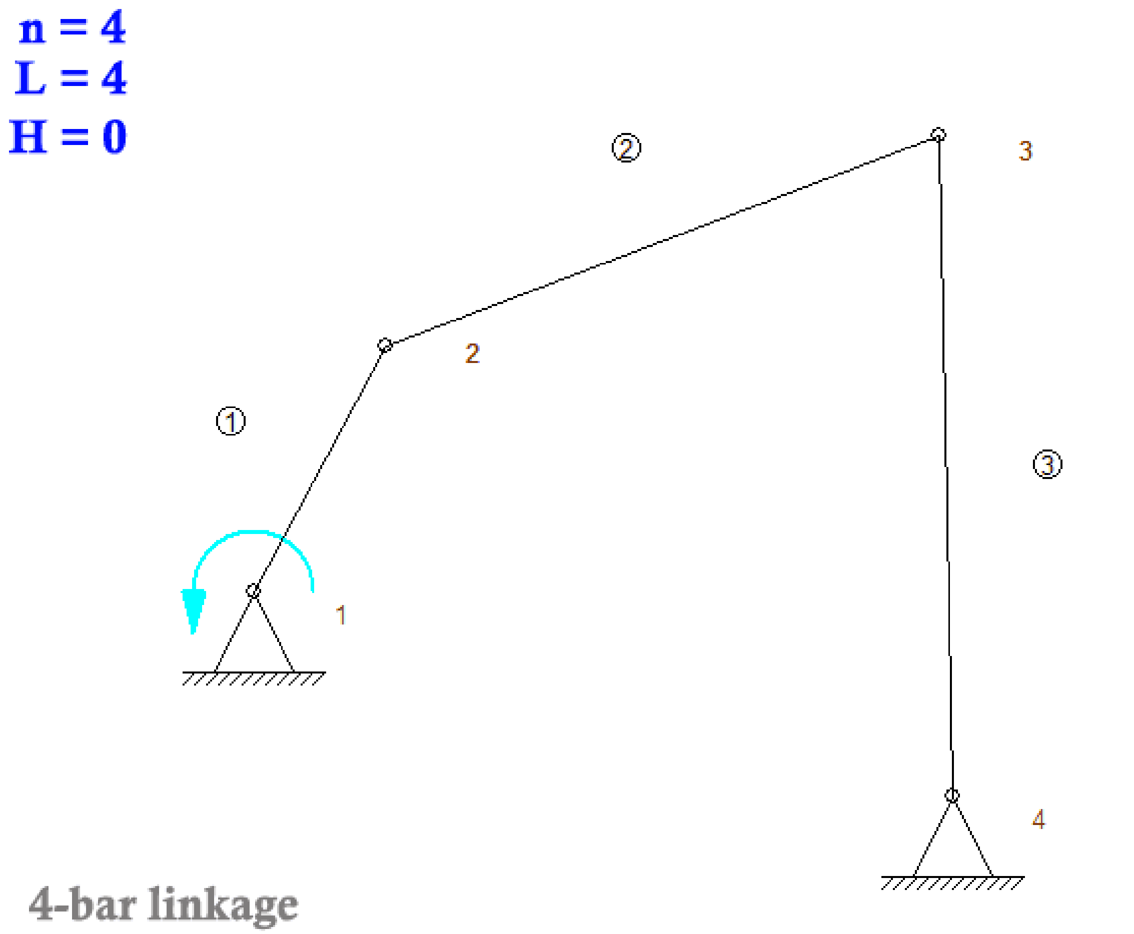
http://ezymechanic.blogspot.com/2015/07/gruebiers-equation-for-calculating.html

Four-bar linkage

$$F = 3(n-1) - 2L - H$$



- $n = 4$
  - *3 links + 1 ground link*
- $L = 4$
  - *4 revolute joints*
- $H = 0$
- $F = 3(4-1) - 2(4) - 0 = 1$



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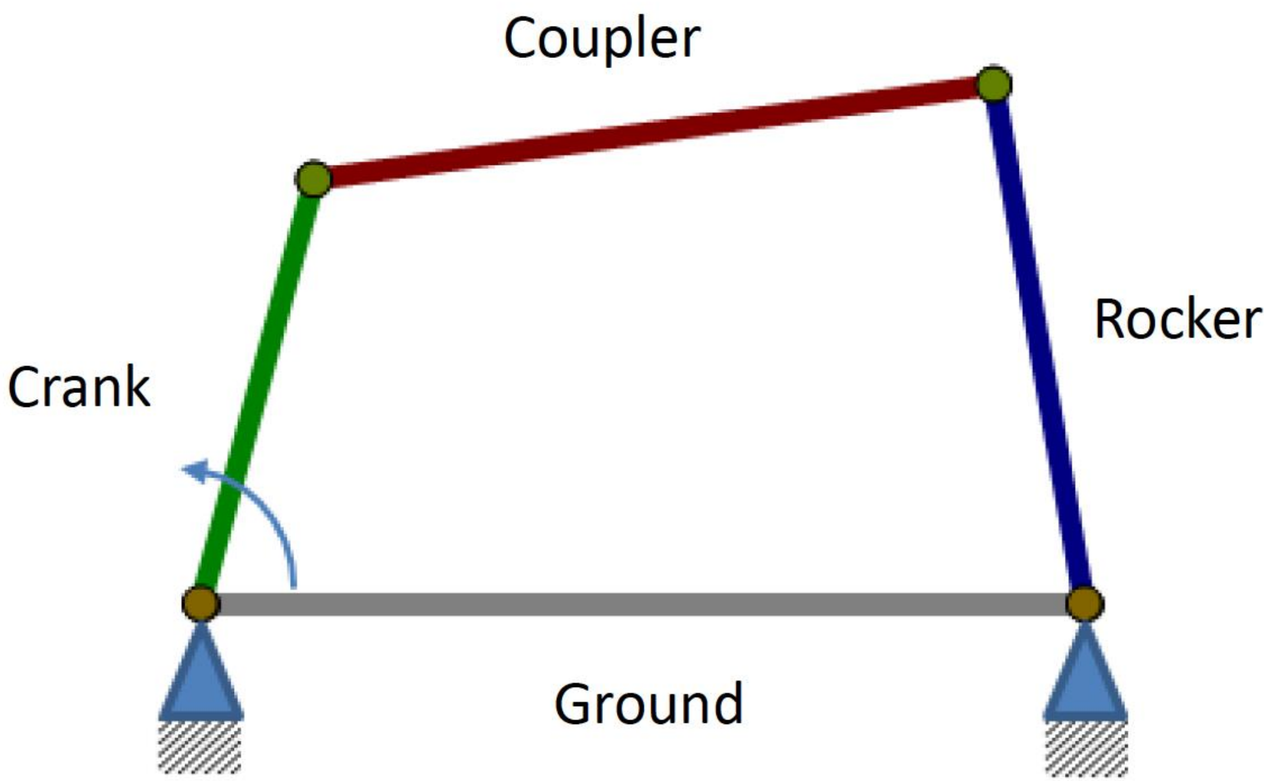
Four-bar linkages

- **Crank:** a link that makes a complete revolution and is pivoted to ground
- **Rocker:** link that has an oscillatory rotation and is pivoted to ground
- **Coupler —(or connecting rod):** a link that has a complex motion and is not pivoted to ground
- **Ground:** any link that that is fixed or not moving

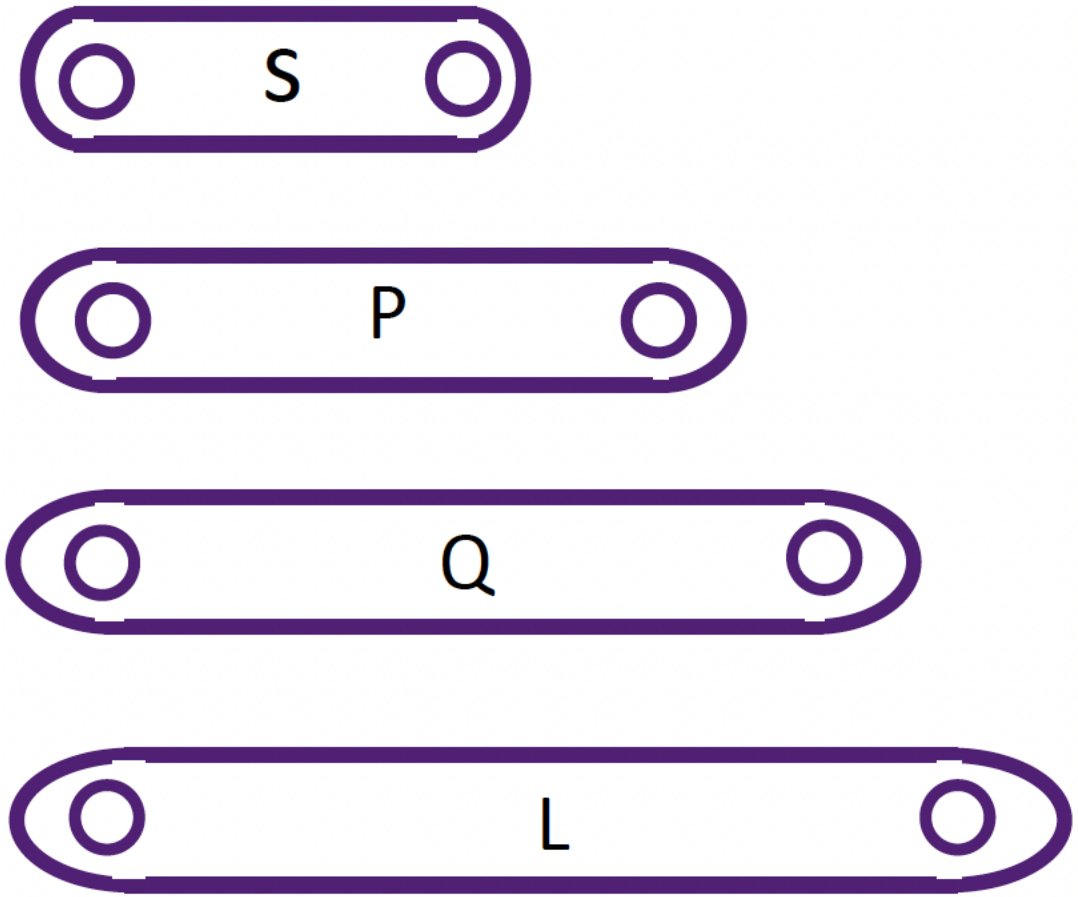


Grashof condition

- Applies only to four-bar linkages
- Predicts behavior based on the lengths of the links
- Independent of the order of link connections
- Independent of which link is grounded

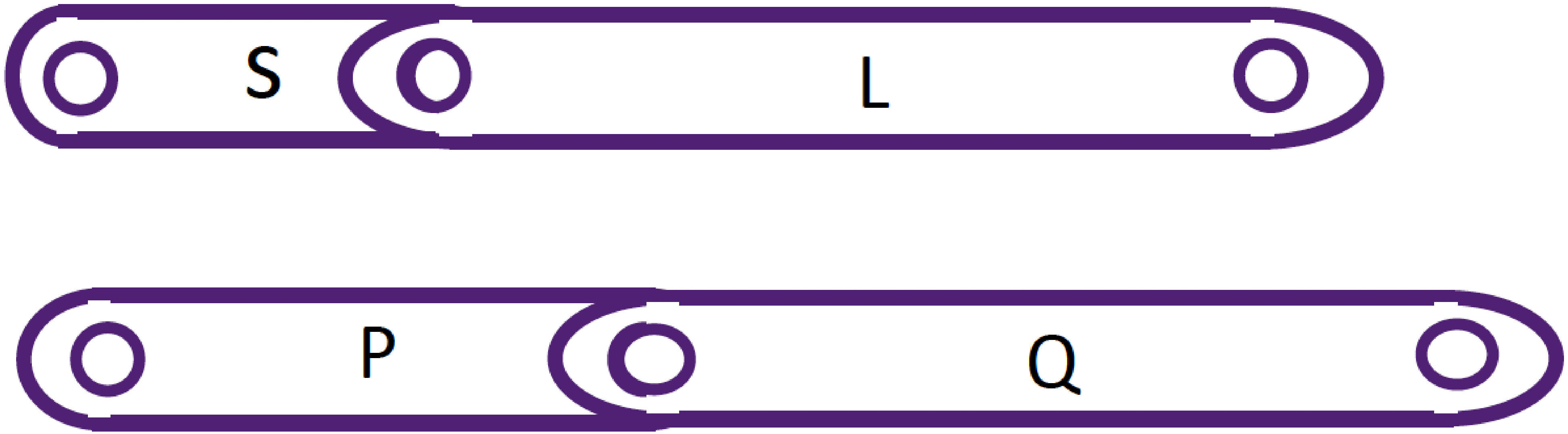
Grashof inequality

- $S + L \leq P + Q$
- If the sum of the shortest and longest link is less than or equal to the sum of the remaining two links, then the shortest link can rotate fully with respect to a neighboring link



by Goonetilleke

Grashof linkage?



by Goonetilleke

How many unique ways can the four bars be assembled in a 4-bar linkage?

- Let's pick bar Q
- Q is connected to two links
- Three remaining bars, taking 2 at a time
  - n = set, r = subset
- Three combinations
  - SQP
  - LQP
  - LQS

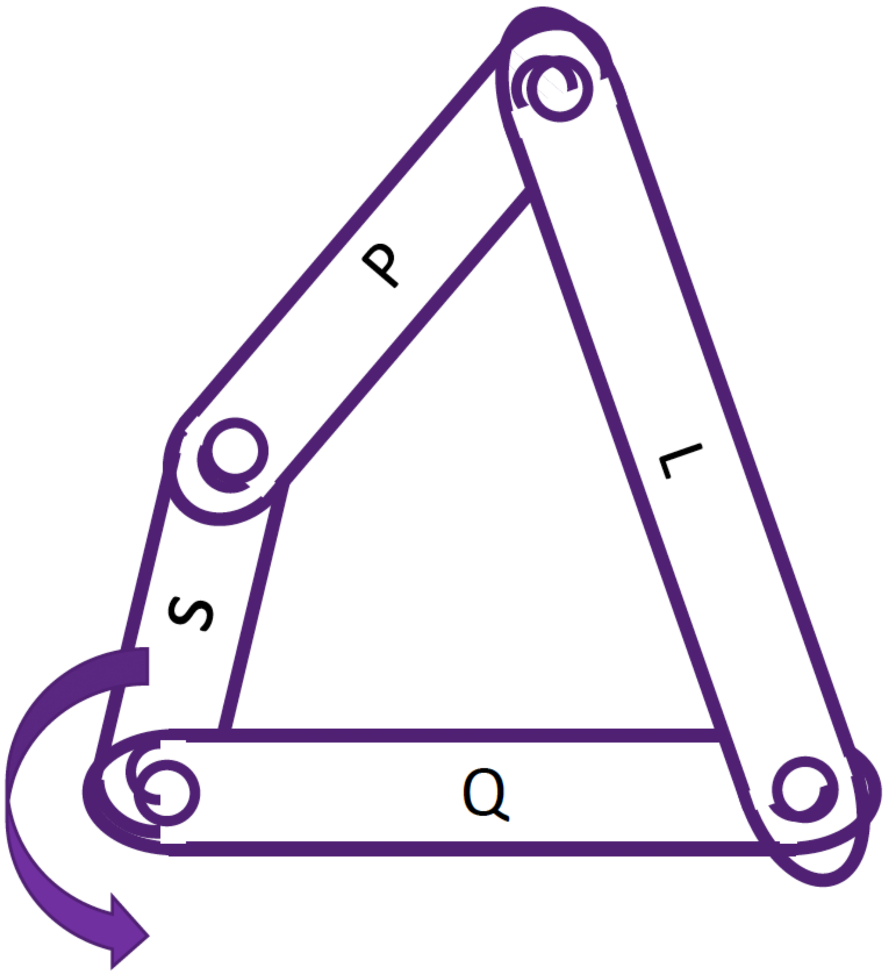
$$C(n, r) = \frac{n!}{r!\,(n-r)!}$$

If Grashof linkage

- One link will make a complete revolution with respect to a neighboring link
- Other links cannot rotate more than 180º
- Depending on which link is grounded, there are three possibilities
  - Crank rocker
  - Double crank
  - Double rocker

Possibility 1

- One end of S is connected to the ground
- Crank rocker
- S will be the crank and the other link pivoted to the ground will oscillate
- Applications?



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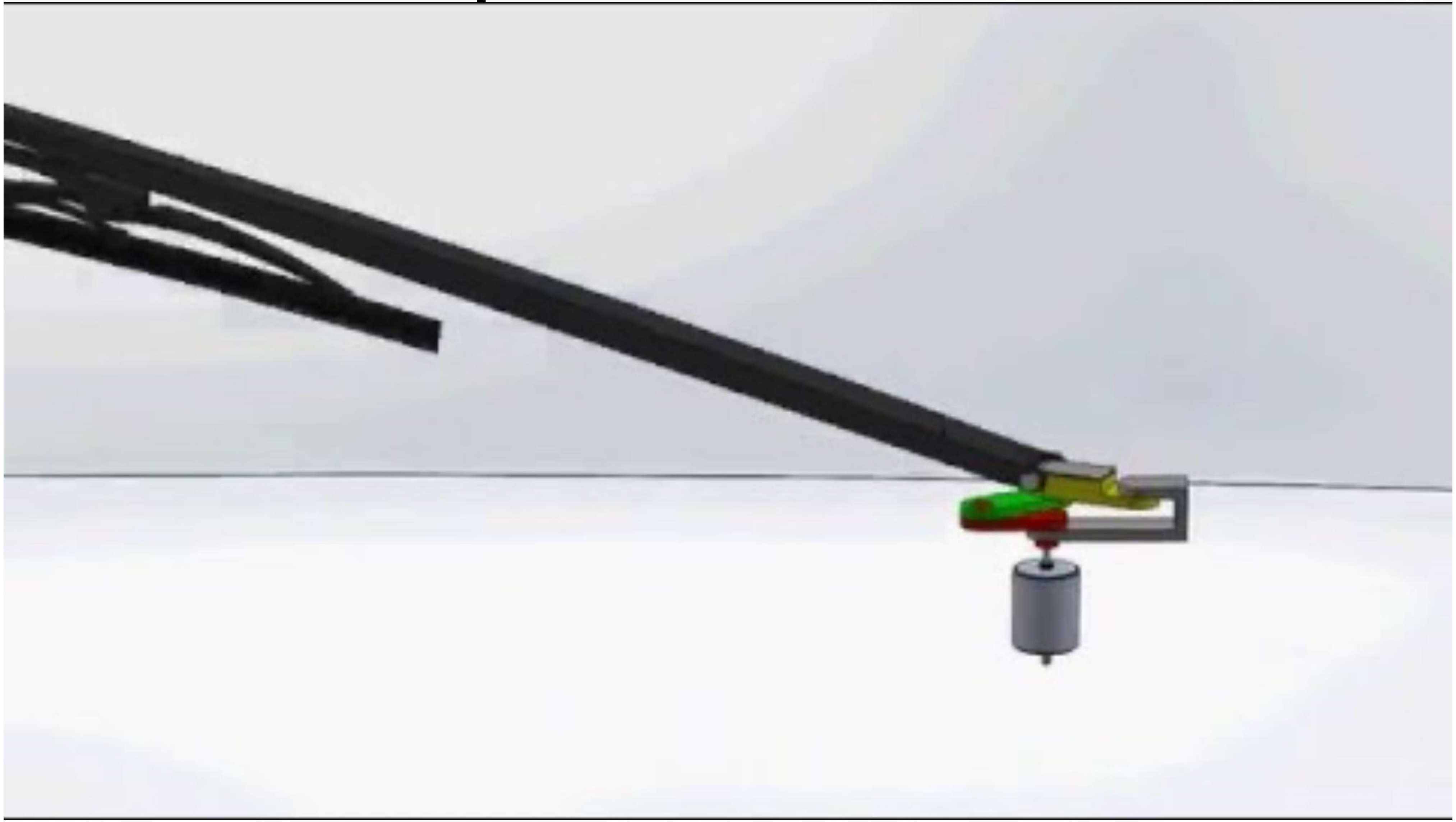
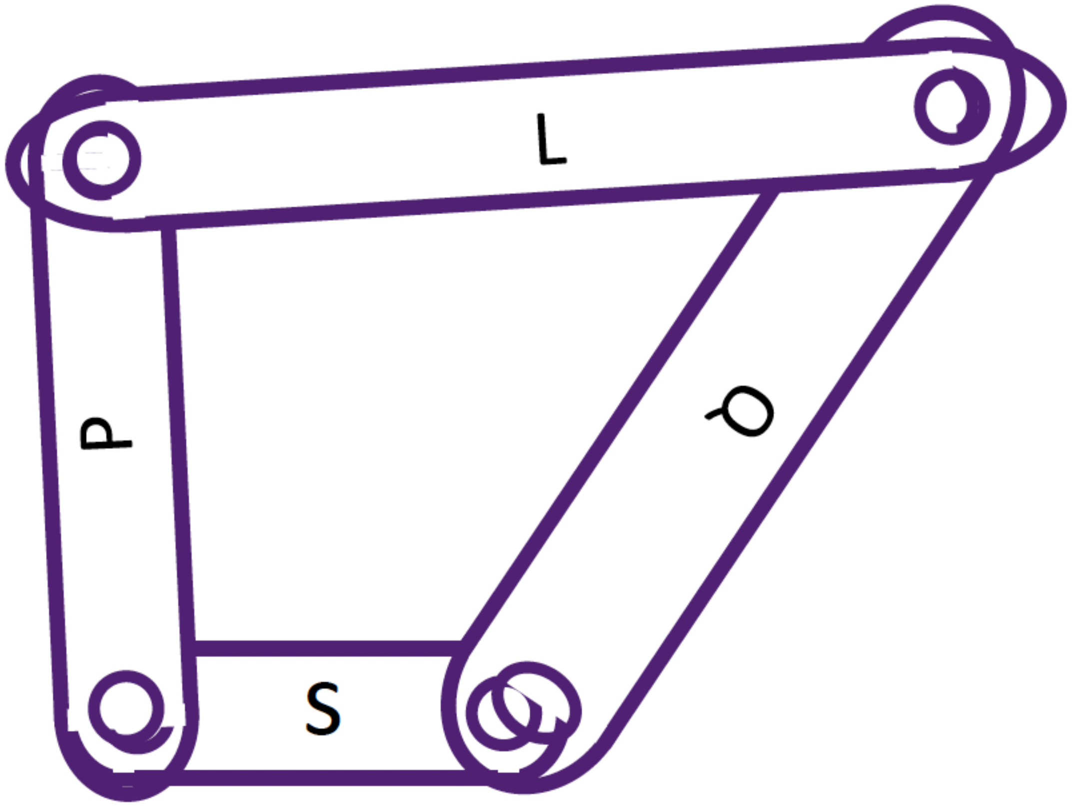
Windshield wiper mechanism



Possibility 2

- S is grounded
- Double crank
- Both links connected to ground will make a complete revolution
- S also makes a full revolution with respect to neighboring links



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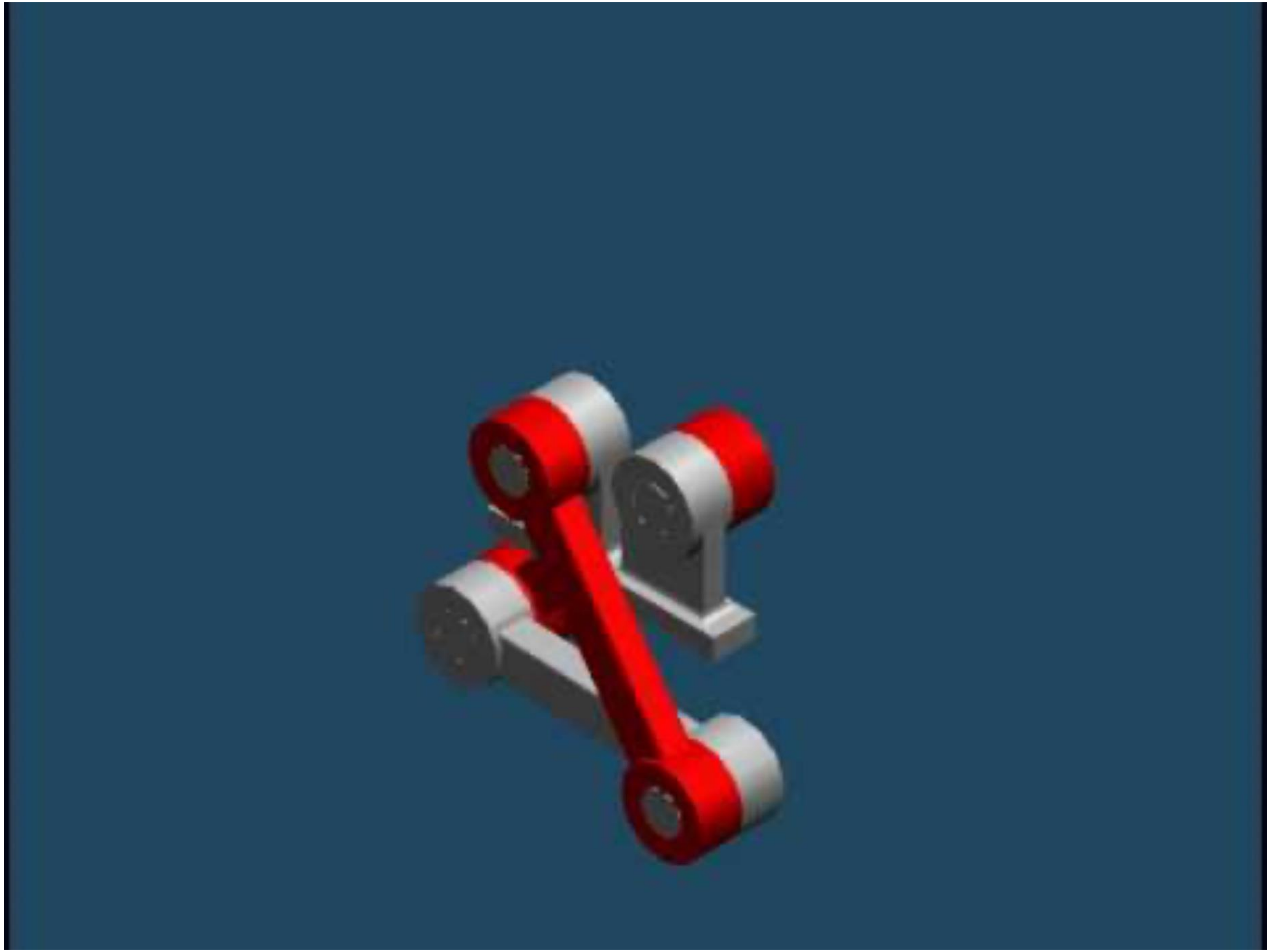
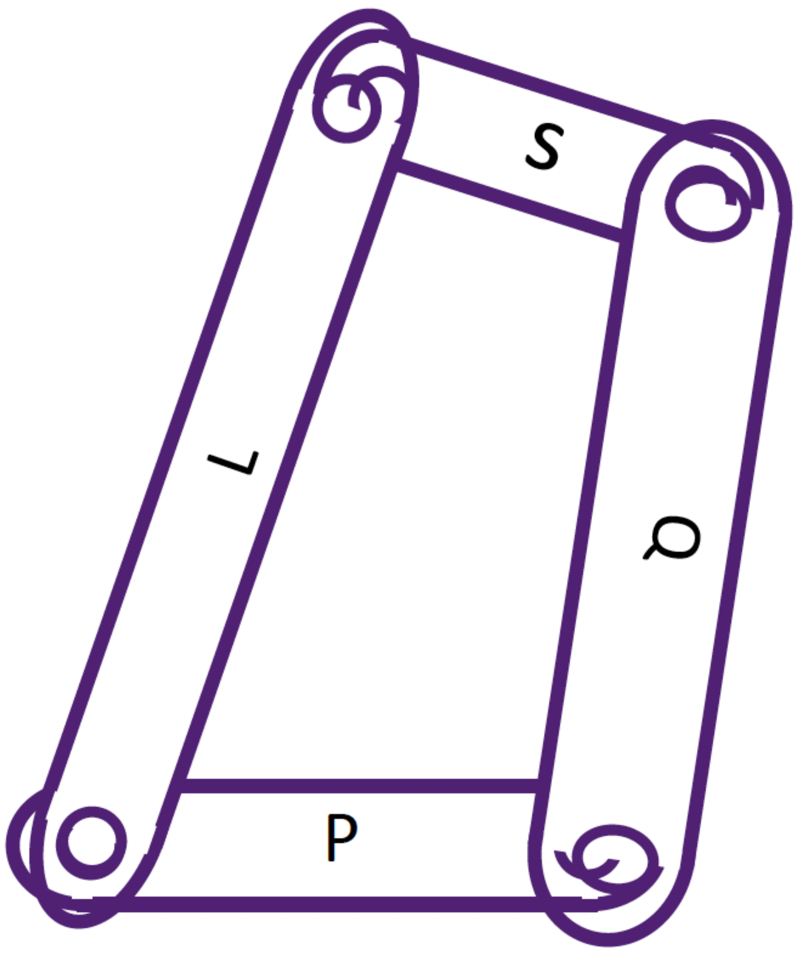


Possibility 3

- Link opposite S is grounded
- Double rocker
- Both links connected to the ground will oscillate



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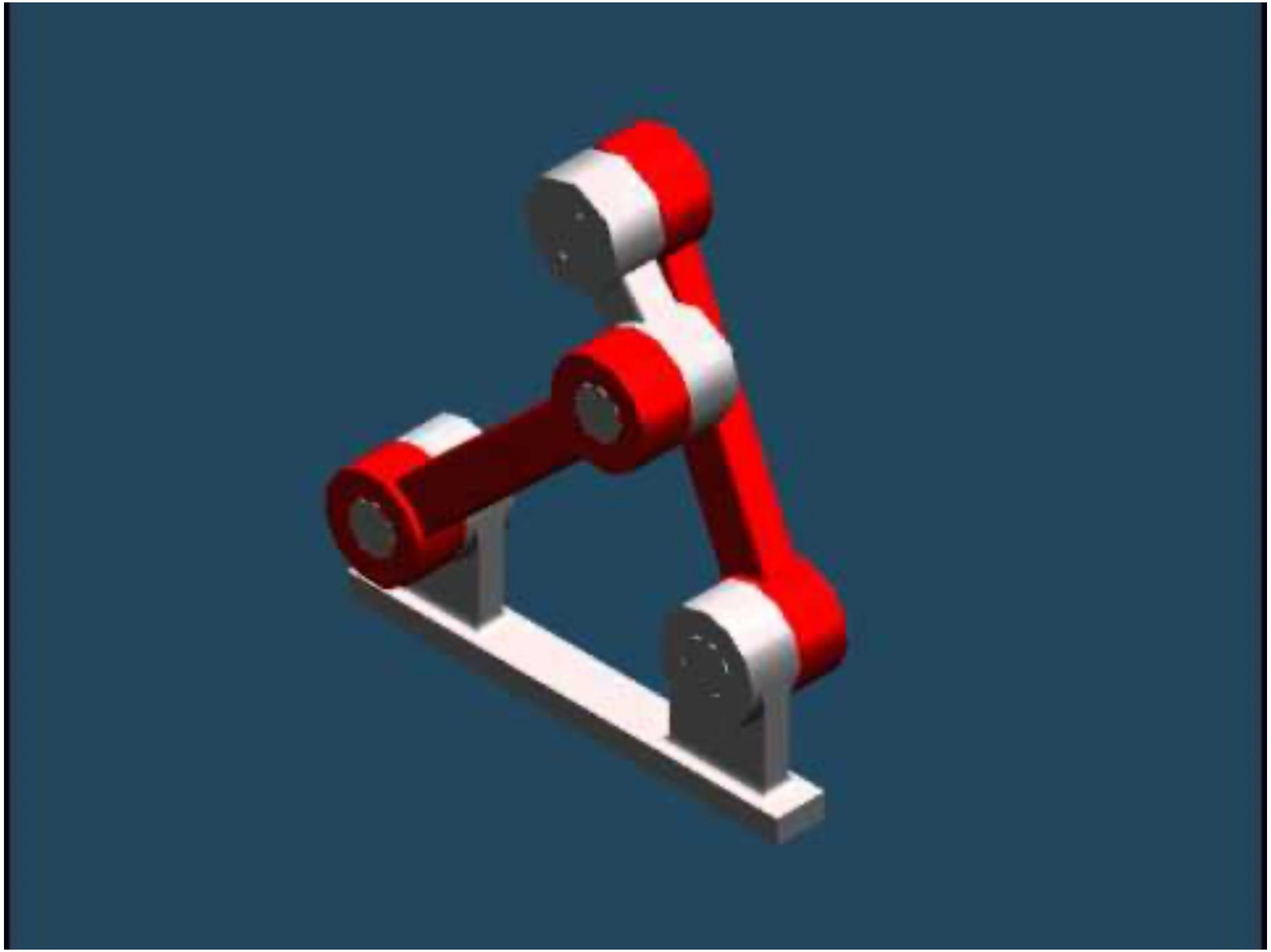
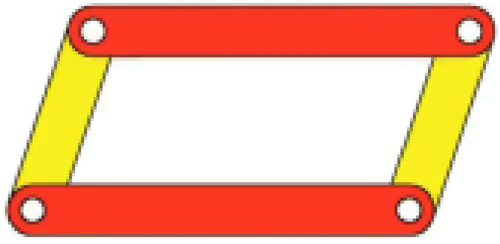
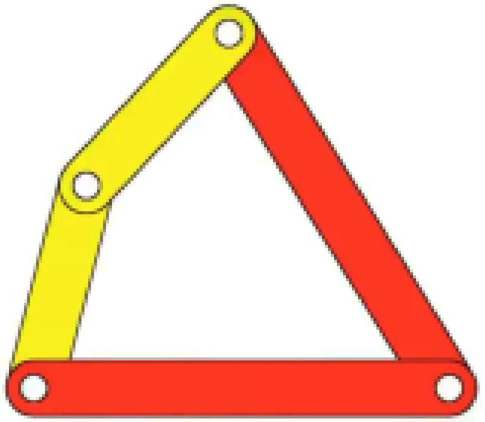


$S + L = P + Q$ (Still Grashof)

- If all four lengths are different, you get the same three conditions as before:
  - crank rocker
  - double rocker
  - double crank
- If P=S then L=Q. Two pairs of equal length
  - Equal links opposite each other --> parallelogram linkage
  - Equal linkages adjacent to each other --> Deltoid linkage. Then crank rocker or double crank.



Parallelogram Linkage



Deltoid Linkage

Non-Grashof linkage

- $S + L > P + Q$
- No link can make a complete revolution
- Links can rotate more than 180°
- All 4 inversions will be triple rockers.

Barker's complete classification of 4-bar linkages

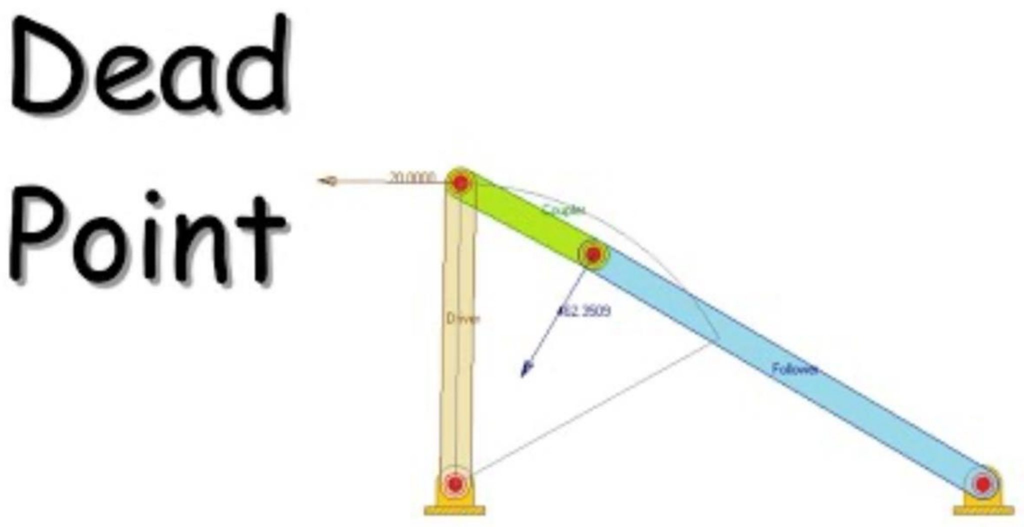
Adapted from ref. (10). s = shortest link, l = longest link, Gxxx = Grashof, RRRx = non-Grashof, Sxx = Special case

Type	$s + l$ vs. $p + q$	Inversion	Class	Barker's Designation	Code	Also Known As
1	<	$L_1 = s =$ ground	I-1	Grashof crank-crank-crank	GCCC	double-crank
2	<	$L_2 = s =$ input	I-2	Grashof crank-rocker-rocker	GCRR	crank-rocker
3	<	$L_3 = s =$ coupler	I-3	Grashof rocker-crank-rocker	GRCR	double-rocker
4	<	$L_4 = s =$ output	I-4	Grashof rocker-rocker-crank	GRRC	rocker-crank
5	>	$L_1 = l =$ ground	II-1	Class 1 rocker-rocker-rocker	RRR1	triple-rocker
6	>	$L_2 = l =$ input	II-2	Class 2 rocker-rocker-rocker	RRR2	triple-rocker
7	>	$L_3 = l =$ coupler	II-3	Class 3 rocker-rocker-rocker	RRR3	triple-rocker
8	>	$L_4 = l =$ output	II-4	Class 4 rocker-rocker-rocker	RRR4	triple-rocker
9	=	$L_1 = s =$ ground	III-1	change point crank-crank-crank	SCCC	SC* double -crank
10	=	$L_2 = s =$ input	III-2	change point crank-rocker-rocker	SCRR	SC crank-rocker
11	=	$L_3 = s =$ coupler	III-3	change point rocker-crank-rocker	SRCR	SC double-rocker
12	=	$L_4 = s =$ output	III-4	change point rocker-rocker-crank	SRRC	SC rocker-crank
13	=	two equal pairs	III-5	double change point	S2X	parallelogram or deltoid
14	=	$L_1 = L_2 = L_3 = L_4$	III-6	triple change point	S3X	square

* SC = special case.

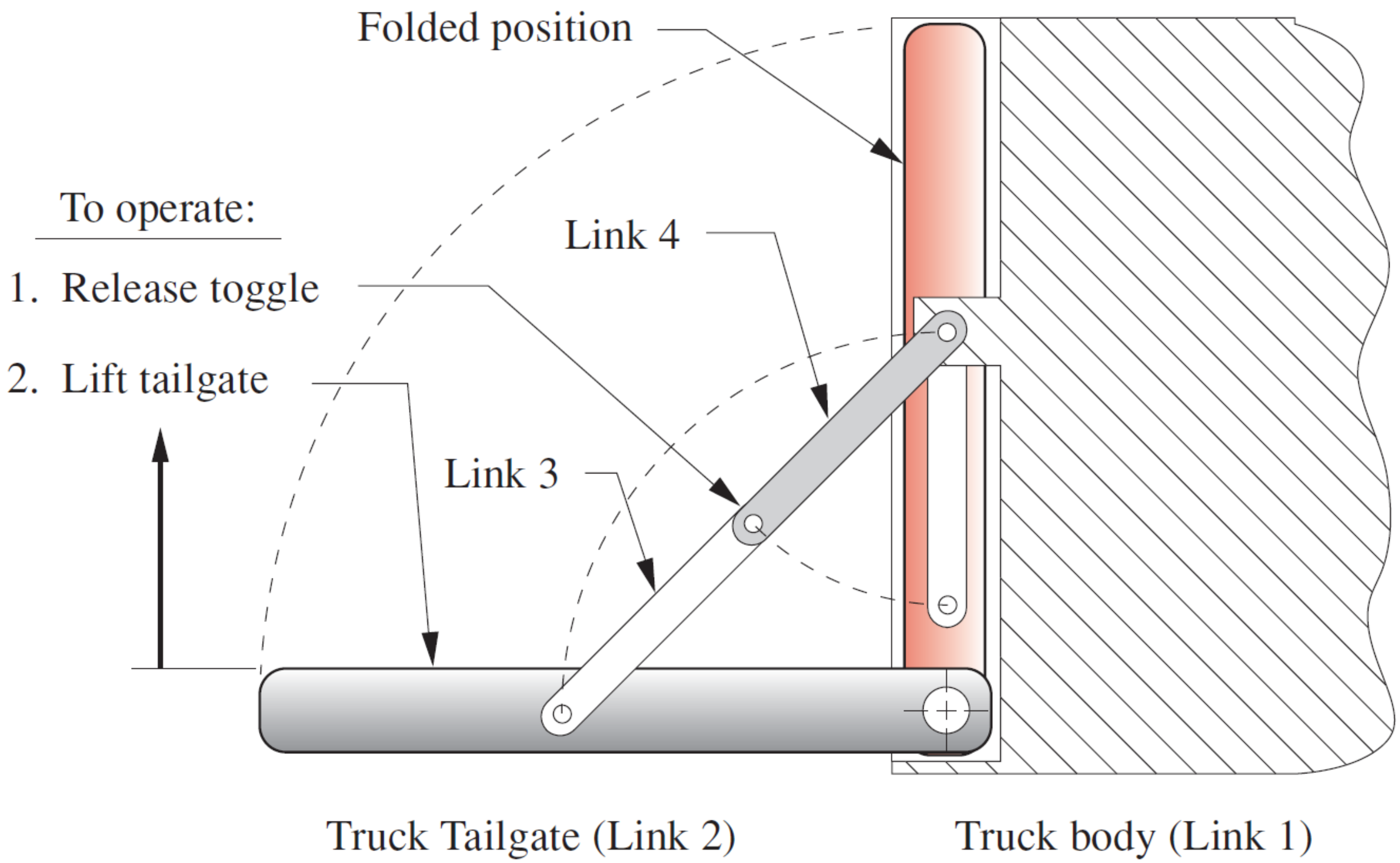
Dead points and change points

- Dead points
  - Alignment of coupler and follower
  - Mechanism gets stuck
- Change points
  - Alignment of all links
  - Indeterminate output: unpredictable behavior



Dead points as a feature?

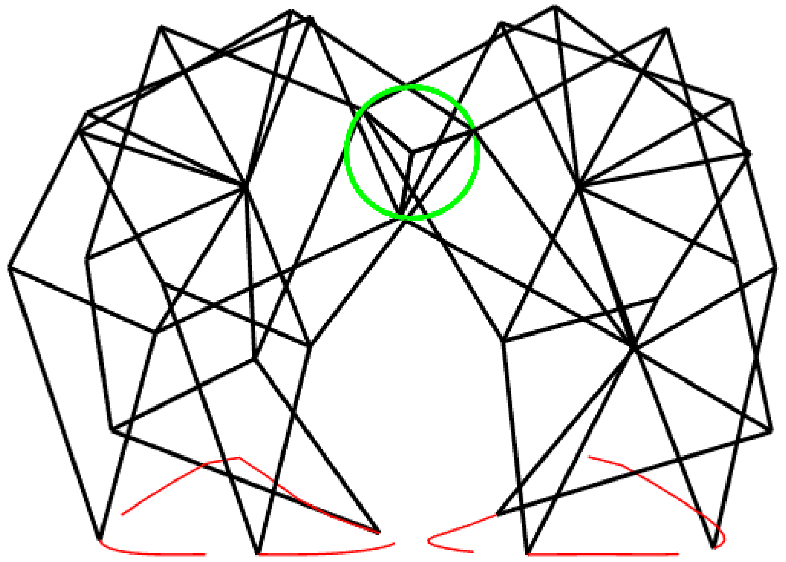
- Self-locking



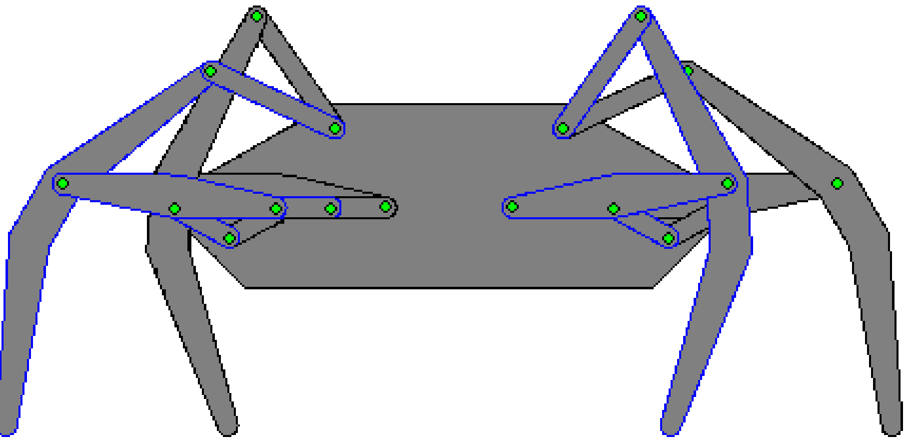
Linkages for legged locomotion

Legged locomotion

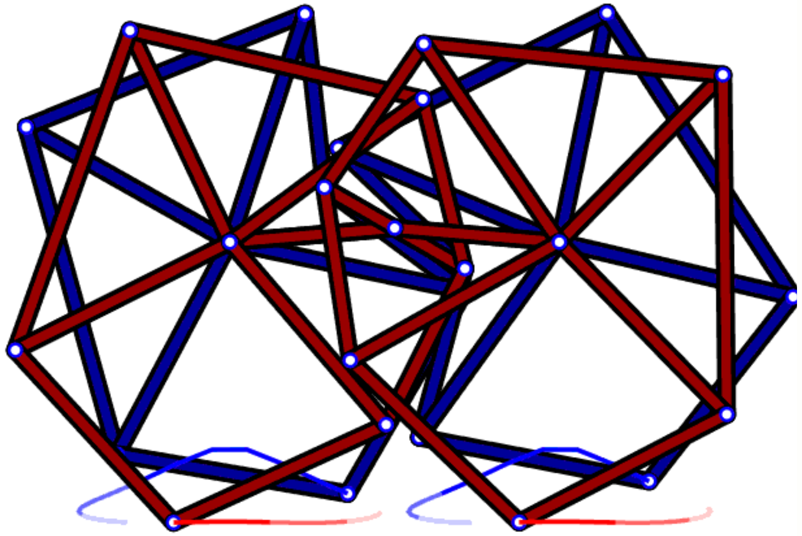
- Intermittent frictional contact with the ground



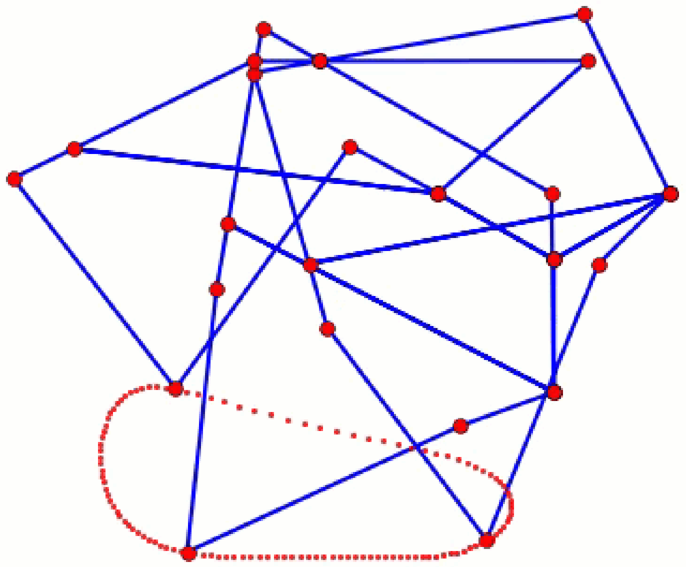
Jansen linkage



Klann linkage



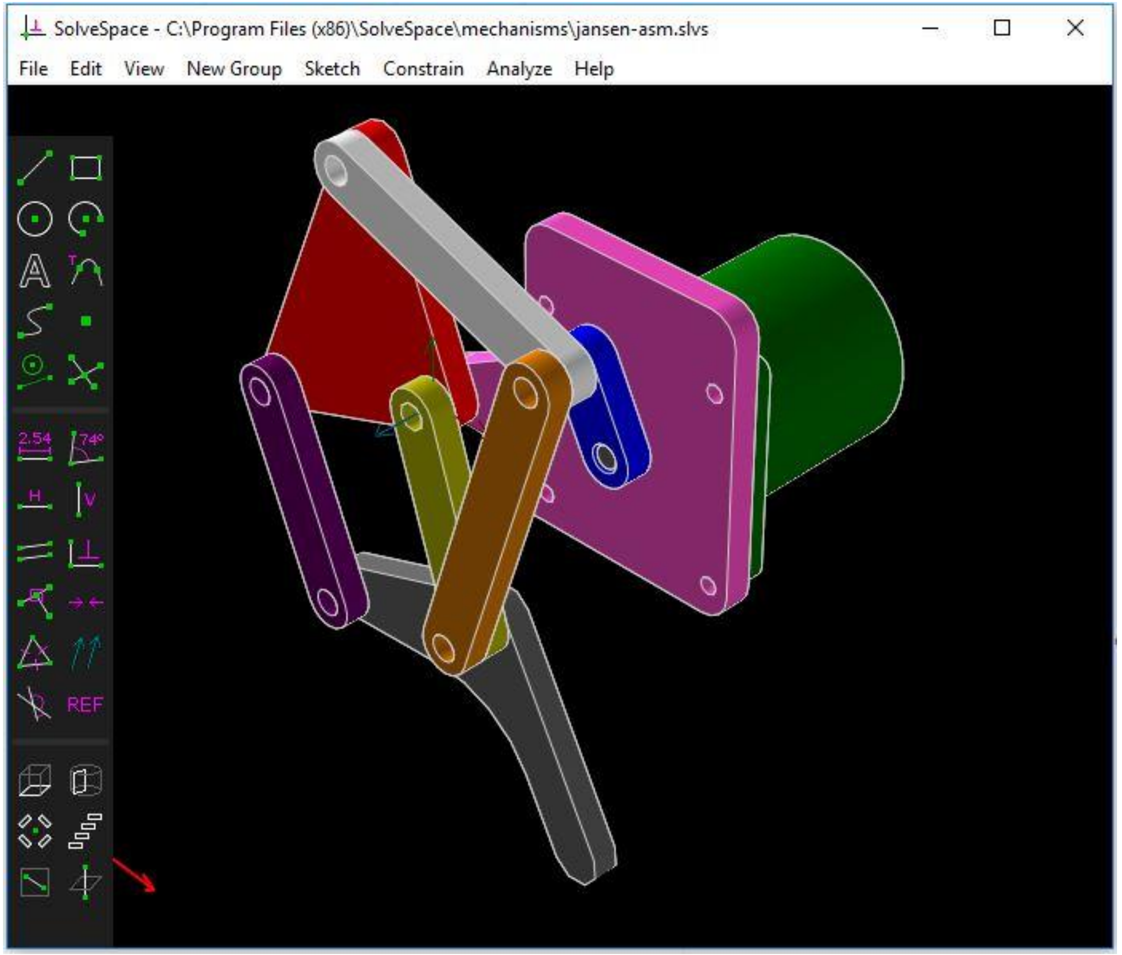
Ghassaei linkage



TrotBot linkage

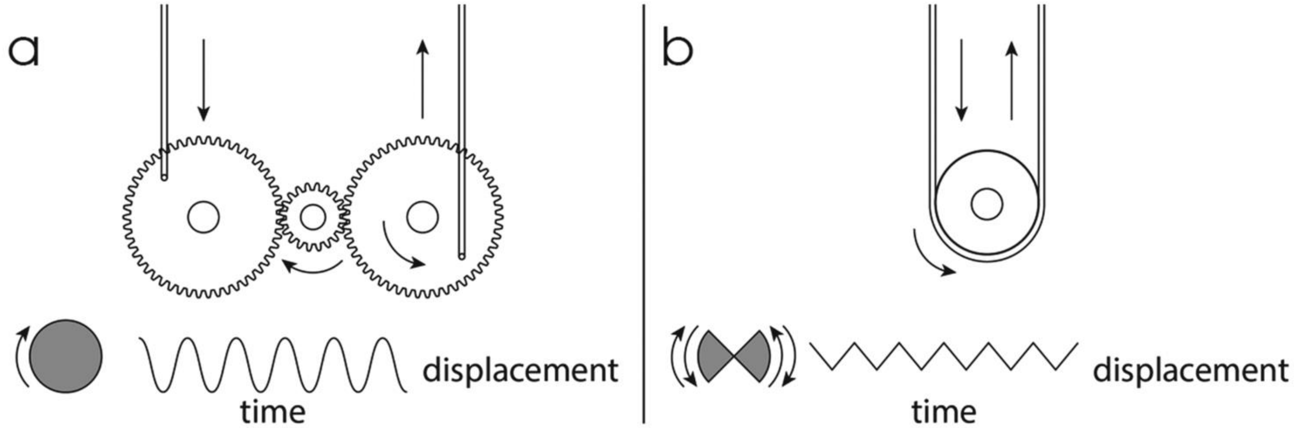
Prototyping software for linkages

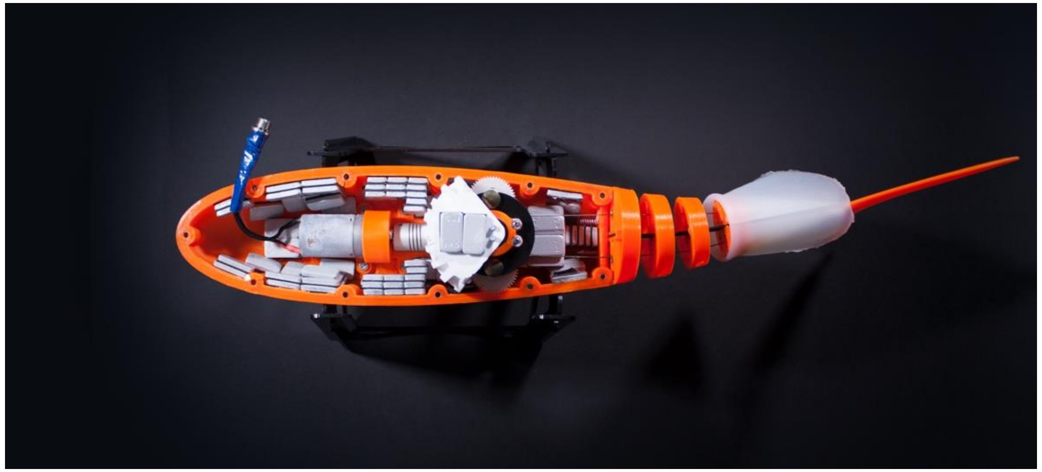
- Fusion 360
- Solidworks
- Artas
- Linkage Mechanism Designer
- SolveSpace
- Algodoo
- MotionGen (IOS and Android)



Robot Fish

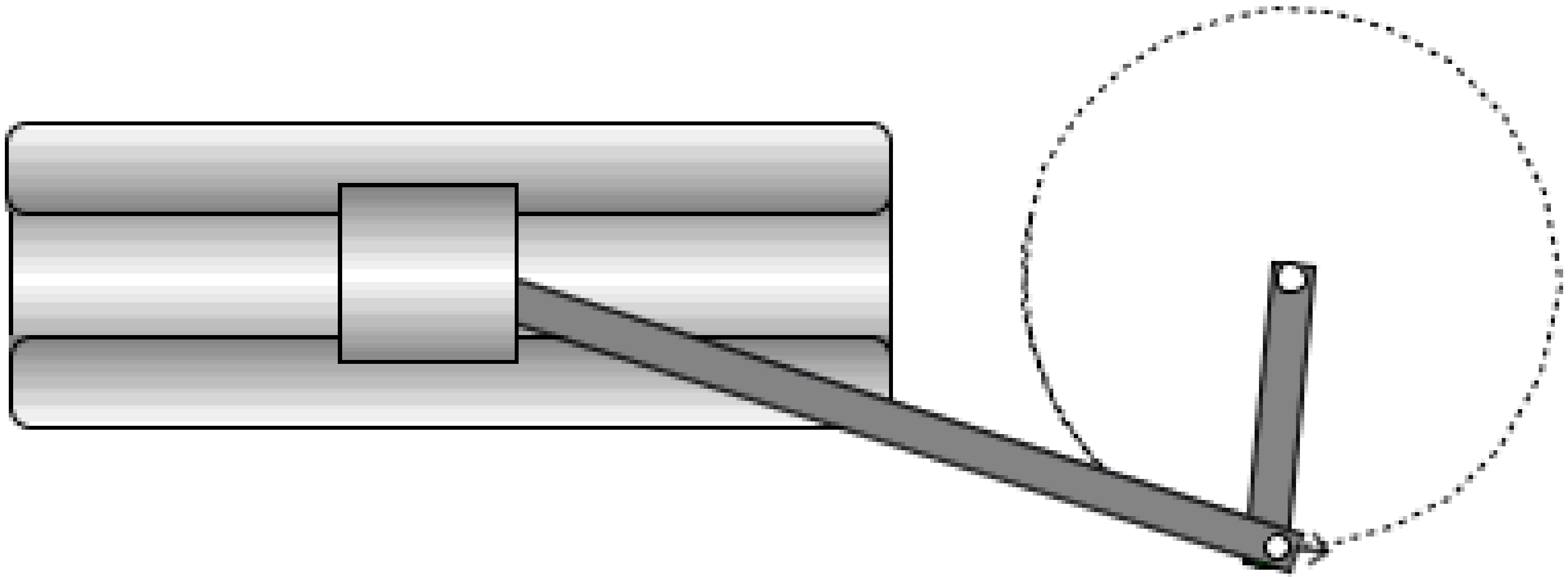
- Rotary motion to reciprocating motion
- OpenFish Mechanism
  - Gearbox with gears spinning in opposite directions
  - More sinusoidal waveform than with servo-motor
  - Higher frequency than with servo-motor



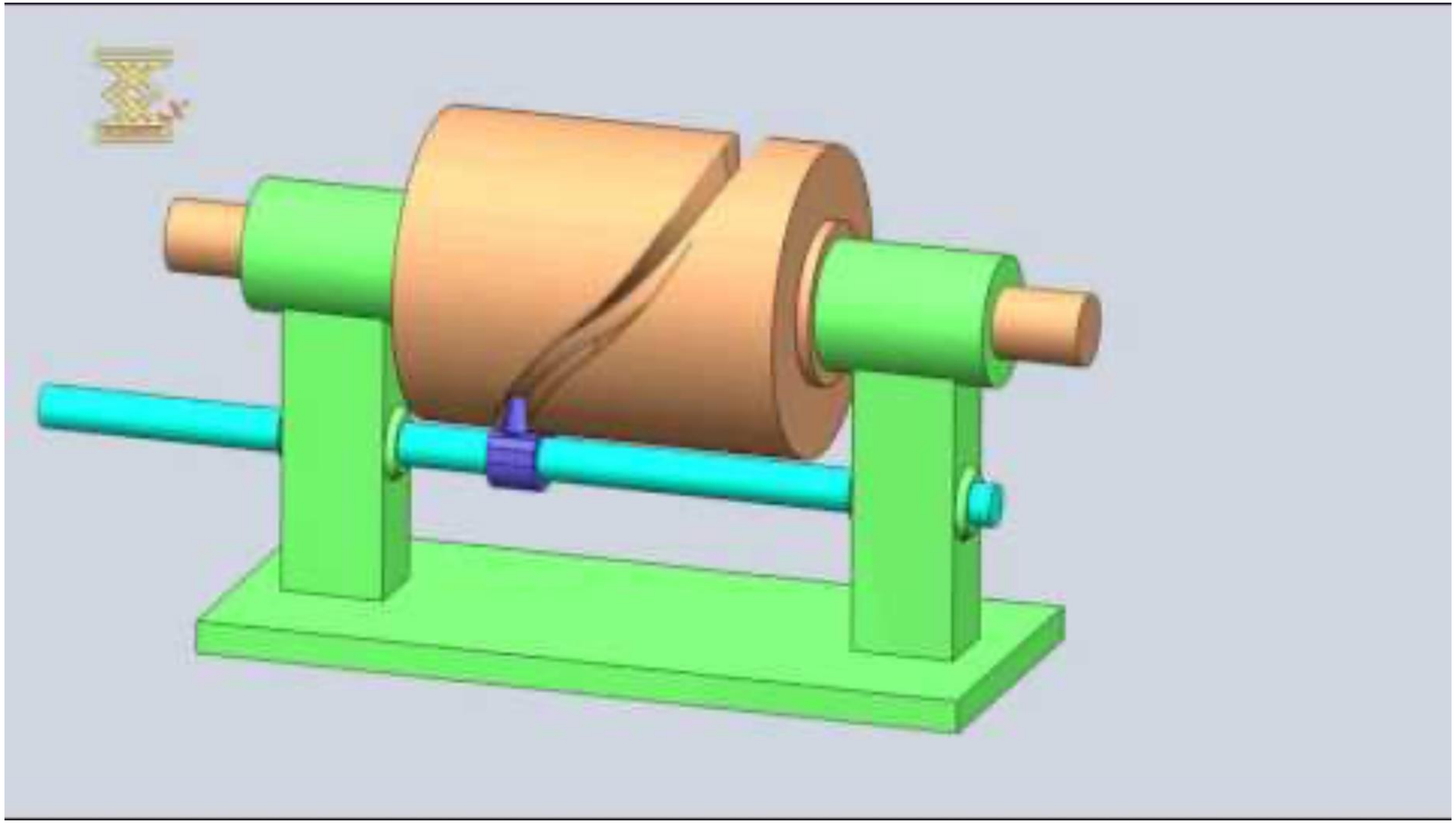


Alternatives: Scotch Yoke

Alternatives: Slider crank

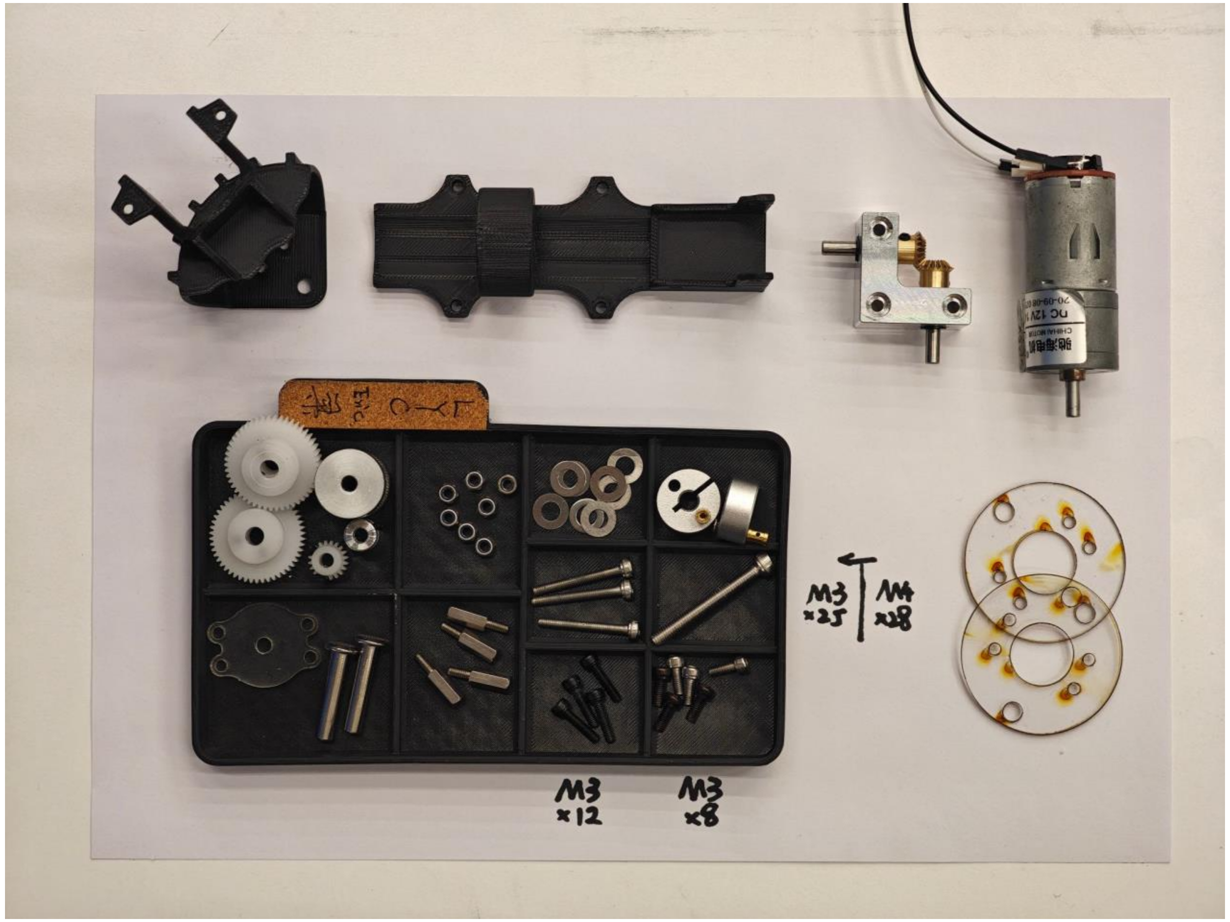


Cylindrical Cam mechanism



Next lab: Assemble the OpenFish propulsion mechanism

- Before the lab:
  - Fabricate all parts listed on the course website



Questions?